

*You have to talk for ten minutes about this subject. Which mathematical notion(s) do you recognize?
The questions may help you, but answering all of them is not compulsory:
you can simply explain a way to solve an exercise, even if you can't find the solution*

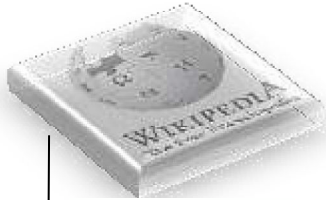
Golden ratio

In mathematics, two quantities are in the **golden ratio** if their ratio is the same as the ratio of their sum to the larger of the two quantities.

Expressed algebraically, for quantities a and b with $a > b > 0$:

$$\frac{a}{b} = \frac{a+b}{a}$$

This equation has one positive solution.

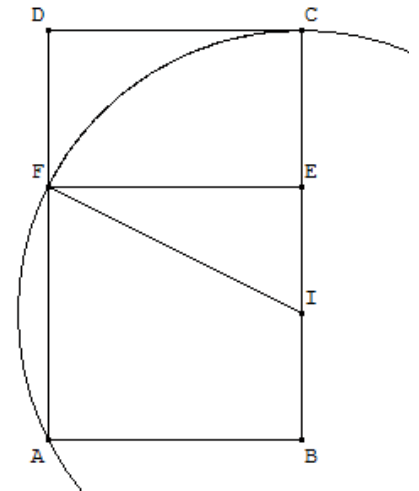


Source : wikipedia

A golden rectangle is a rectangle in which the ratio of the longer side to the shorter is the golden ratio.

- 1) The golden ratio is a famous number. What can you say about it ?
- 2) The golden ratio is the positive solution to the quadratic equation $x^2 - x - 1 = 0$. Solve the equation and find the value accurate of the golden ratio.
- 3) In the figure on the right, ABEF is a unit square.
I is the midpoint of BE.
IF is the radius of the arc that defines point C.

Prove that ABCD is a golden rectangle.





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The race

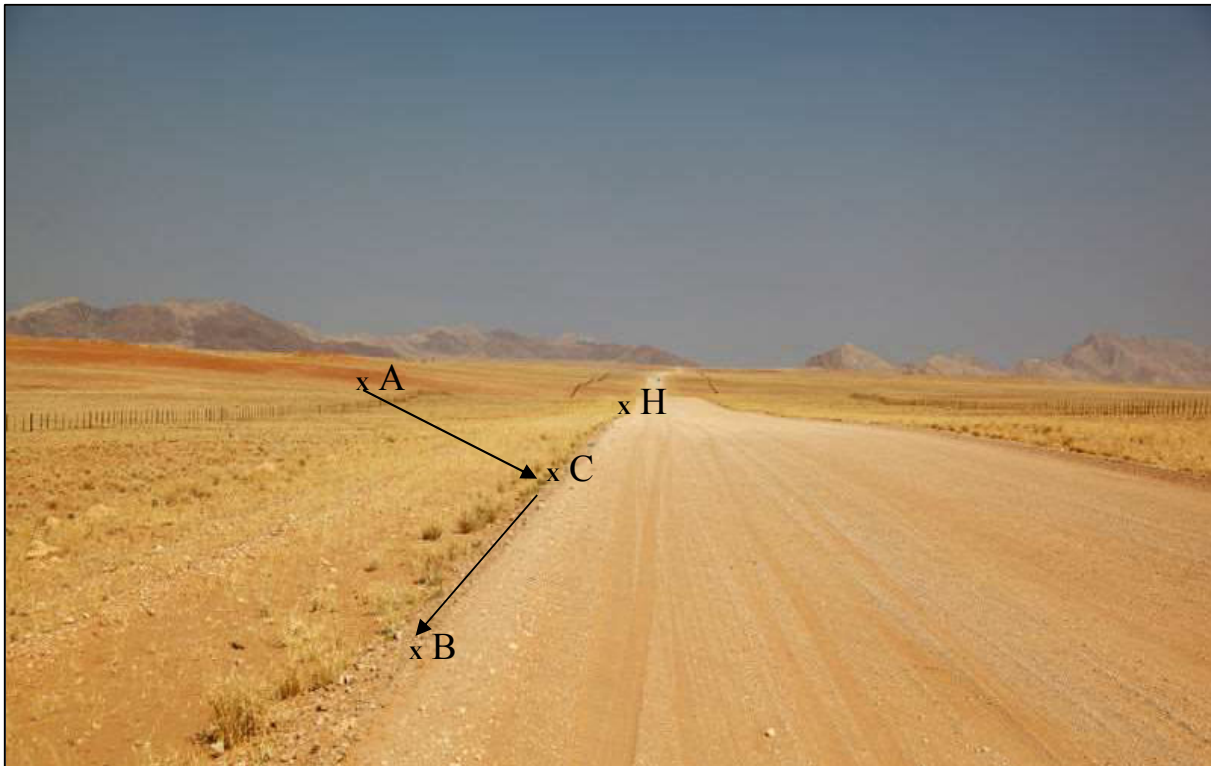
While racing through the rock desert at the speed of 25mph, the driver of a rally car catches sight* of a road. The driver is located at point A. The shortest distance AH between point A and the road is equal to 15 miles.



The driver wants to reach point B on the road 27 miles away from point H.

Draw the situation seen from above.

Knowing that he will be able to drive 80mph on the road, at which point, named C, should the driver join the road to minimize his travel time?



(Figure not to scale)

To catch sight : *apercevoir*

This function could help you : $f(x) = \frac{\sqrt{15^2+x^2}}{25} + \frac{27-x}{80}$.

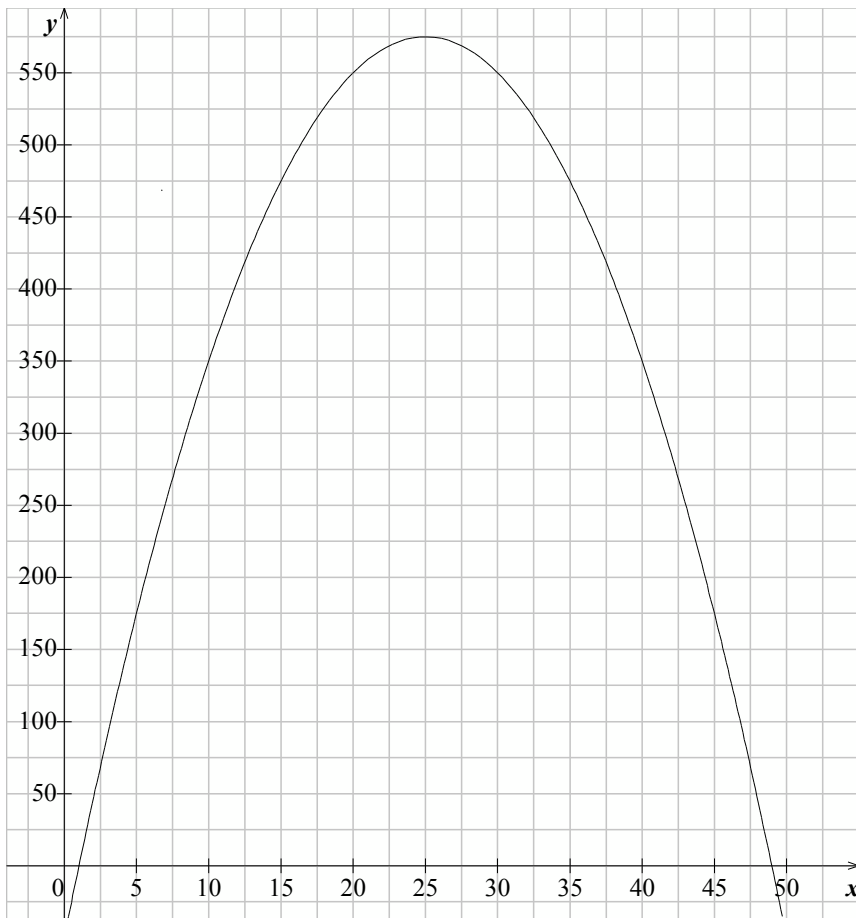
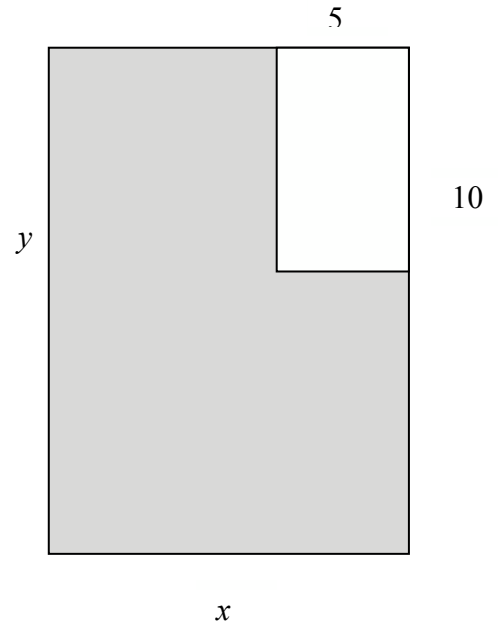
If your calculation seems too long, use your calculator!

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A shape of metal

A small rectangle is cut off from a rectangular metal shape, as shown below.
All the measurements are in feet. The perimeter of the final shape is 100 feet.

- 1- What is the smallest value of x ?
- 2- Prove that $y = 50 - x$
- 3- Show that the area of the final shape can be expressed in terms of x by
 $A(x) = -x^2 + 50x - 50$
- 4- The curve of the function A is sketched below. What can you infer from it?



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An addictive game...

Tom is playing a game in which he has to get rid of the pigs by throwing birds.
We will admit that the trajectory of the bird is a parabola.
You can see below Tom's first throw (the units on the axis are in meters and the bird starts at the origin).



Introduction:

The functions below can obviously not model the trajectory of the bird. Why? Explain in a few words.

$$f_1(x) = -0.1x^3 + 0.2x^2 + 0.4x$$

$$f_2(x) = -0.2x^2 + 0.7x + 0.1$$

$$f_3(x) = 0.1x^2 - 0.7x$$

Questions:

We will now admit that the trajectory can be modelled by this function:

$$f(x) = -0.14x^2 + 0.7x$$

Tom's goal was to hit the blue stone on the left. Is he going to succeed?
What is the highest point the bird can reach? When does it occur? What happens at this moment?
If it hadn't touched anything, how far from the origin would the bird have landed?



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Ski jump



The ski jump is divided into four separate sections:

- 1) In-run
- 2) Take off (jump) at point T
- 3) Flight
- 4) Landing at point L

Introduction :

A, B and L are collinear points.

$A(0, 50)$; $B(40, 24)$.

Find an equation of the line that connects the point A to the point B.

This line is called the landing slope.

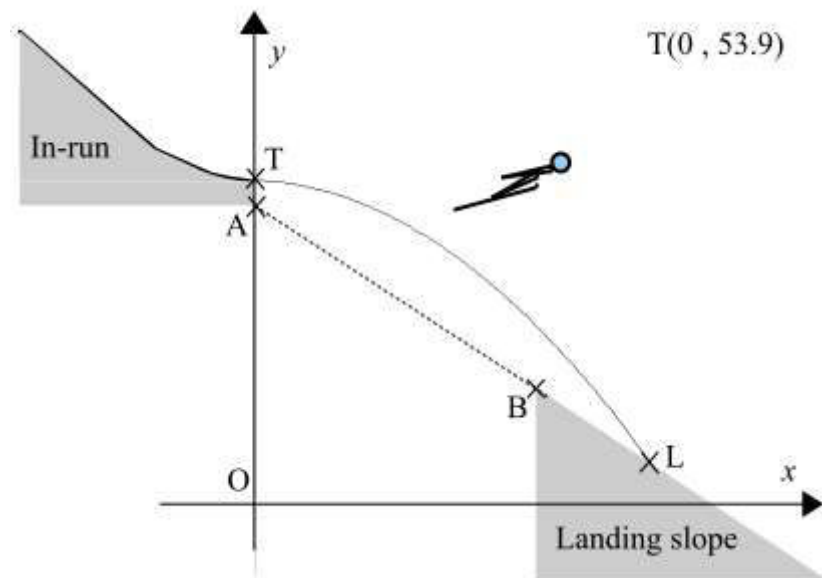
Questions :

We admit that the jump has a parabolic shape.

Its equation is :

$$y = -0.00825x^2 - 0.12x + 53.9$$

where x and y are measured in metres.



- 1) Find an equation of the tangent to the curve that goes through the point $T(0, 53.9)$.
- 2) Calculate the coordinates of the point L where the ski jumper lands. (Round the coordinates to one decimal place).
- 3) Knowing that $T(0, 53.9)$, calculate the length of the jump (distance between points T and L) and round it to one decimal place.

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A box of chocolates



A box of chocolates contains 20 chocolates, all of which are either hard or soft centred. Some of the chocolates contain nuts. 13 chocolates have hard centres, of which 6 contain nuts. There are 10 nutty chocolates in total.

- 1) Represent the data.
- 2) A chocolate is selected at random. Find the probability of :
 - (i) it having a soft centre.
 - (ii) it having a hard centre, given that it contains a nut.
- 3) Now, our main interest is to know if the chocolate is hard or soft centred. 2 chocolates are selected at random without replacement.
 - (i) Draw a tree diagram
 - (ii) Find the probability that at least one chocolate has a hard centre.
 - (iii) What is the probability that the first chocolate is soft centred, given that the second one is hard centred?

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Security

John Brown has equipped his house with an alarm system which is designed to go off if a burglar breaks into the house. But it may not work properly.

Some tests have shown that, in one day:

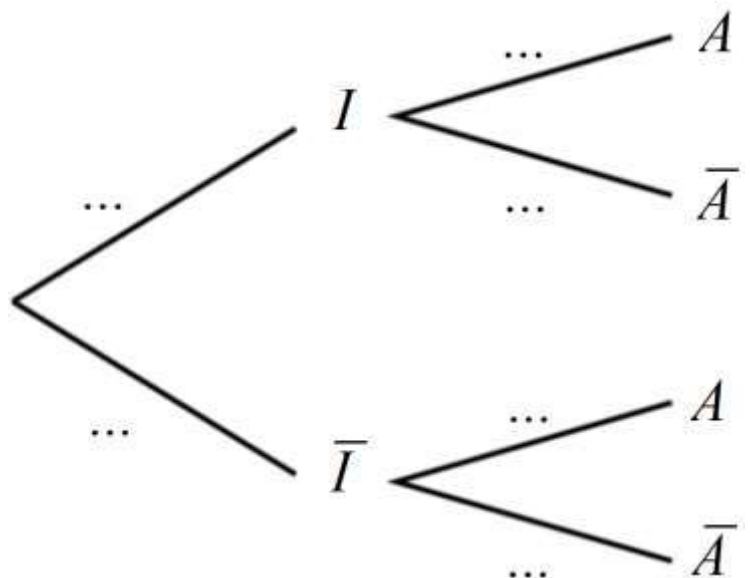
- The probability that the alarm system goes off without any reason (there's no burglar!) is equal to $\frac{1}{80}$
- The probability that a burglar breaks into Mr Brown's house without the alarm going off is equal to $\frac{1}{800}$
- The probability that an incident happens is equal to 0.01.



Let A be the event: “ the alarm system goes off” and let I be the event : “one incident happens”.

Let \bar{A} and \bar{I} be the complements of A and I .

- Fill in the tree below and specify the probabilities on each branch of the tree:
 - Deduce the probability $P(A)$ that the alarm system goes off.
- What is the probability $P(B)$ that, one day, the alarm system doesn't work properly?
- Given that the alarm system has just gone off, what is the probability that an incident has really happened?





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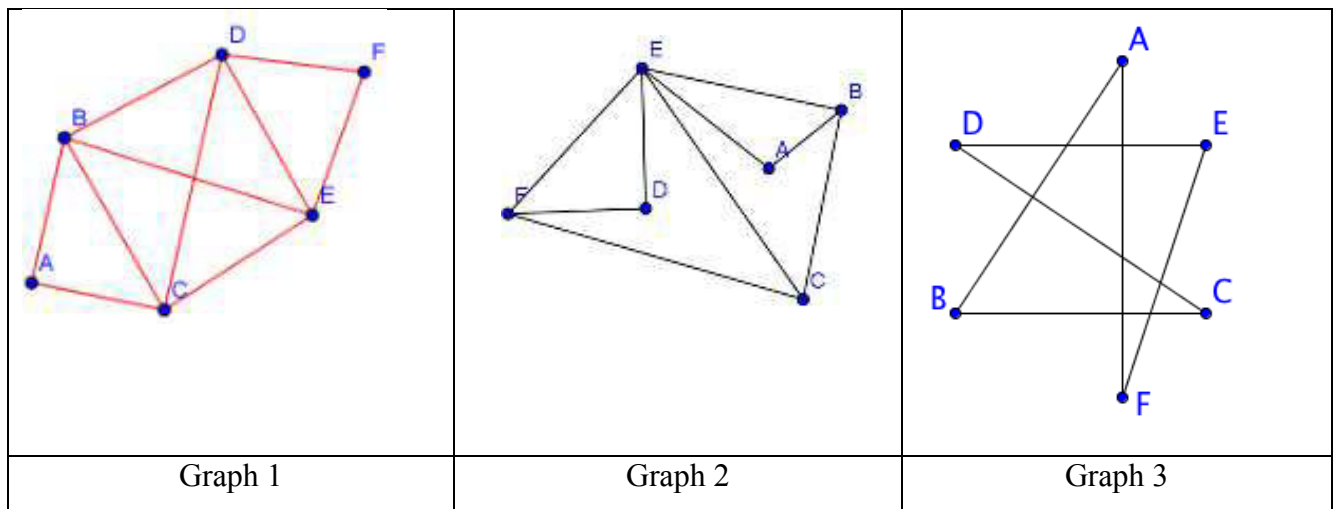
Tropical fishes

A tropical fish hobbyist has six different types of fish: **Alphas, Betas, Certas, Deltas, Epsalas, and Fetas**, which will be designated by **A, B, C, D, E, and F**, respectively. Because of predator-prey relationships, water conditions, and size, some fish must not be kept in the same tank. The following table shows which fish must not be together:

Type	A	B	C	D	E	F
Must not be with	B, C	A, C, E	A, B, D, E	C, F	B, C, F	D, E

We want to use a graph describing the situation.

- 1) What are the vertices?
- 2) What are the edges and how many edges are there?
- 3) Explain why none of the following graphs describe the situation.



- 4) Draw the suitable graph.
- 5) What is the smallest number of tanks needed to keep all the fish? Explain your answer.



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A bike tour

Every year, « The Five Boro Bike Tour » offers more than 30,000 cyclists the opportunity to ride around New York city through all five boroughs : Manhattan, The Bronx, Queens, Brooklyn, and Staten Island through streets, tunnels and bridges totally free of traffic.

The following map shows the five boroughs of New York City and how they are connected by major tunnels, bridges and a ferry route.



1. Draw a graph to summarize the situation.
2. Give an example of chain going through every vertex.
What do you call a graph with this characteristic?
3. If you were in charge of planning the trip using every route marked on the map once and only once, where would you place the starting and finishing lines? Give an example of such a trip.
4. Do you think that it is possible for the cyclists to start and finish the tour at the same place in New York city using every route marked on the map once and only once? Explain why or why not.
5. What are the changes you would have to make for such a trip to be possible?
6. You are thinking of giving every person in charge of safety a high-visibility jacket.
As the cyclists go from one borough to the next, the color should change.
However, the total number of colors used should be kept to a minimum number to reduce the cost.
Give an example of a valid choice.